COT 3100 In-class Exercise 6

Name: USF ID:

**Problem 1 Prove statement by contradiction**

1. For any integer, if then.

Proof by contradiction:

Suppose not. That is suppose there is an integer such that and. [We must deduce a contradiction].

By the quotient-remainder theorem, can be written in the form for some integer. This will give us, meaning is not an integer.

However, should be an integer since the products and difference of integers are integers and 3, and are integers. This is a contradiction. Therefore, if then.

1. If and , then not divides or not divides.

Proof by contradiction:

Suppose not. That is suppose there are integer and with such that and By definition of divisibility, and for some integers and. Subtracting one equation from the other gives, so. Since is positive, is also positive (otherwise would be negative). Then is a positive integer and, so. Thus we have and, This is a contradiction. So, not divides or not divides.

**Problem 2 Prove statement by contraposition**

1. Let . If is even, then is odd.

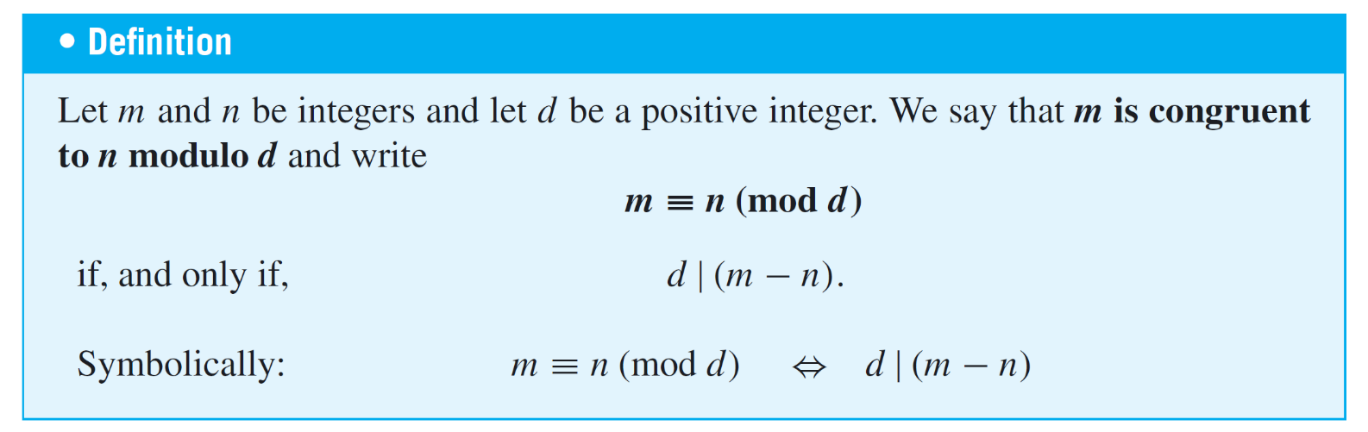
Proof by contraposition:

Suppose thatis even. [We want to show that is odd.] By definition of odd, for some integer.

by substitution

by algebra

Let . is an integer since the products, sums and differences of integers are integers and 2, 6 and are integers. So, for some integer. By definition of odd, is odd [as was to be shown].



**Problem 3:** Prove the statement: let is a positive integer. The integers *m* and *n* are congruent modulo if and only if there is an integer such that .

Proof the statement: if then

Suppose and are any integers, and is a positive integer, such that. By definition of congruence modulo, we have . Thus, by definition of divisibility, for some integer. Adding *n* to both sides gives that.

Proof the statement: if, then

Suppose and are any integers, and is a positive integer, such that for some integer. Subtracting to both sides gives that , where is an integer. By definition of divisibility,. Therefore, by definition of congruence modulo .